Teaching to Solve.

"Yes, the solution seems to work, it appears to be correct; but how is it possible to invent such a solution?" (G. Polya, 1948)

The writing on effective instruction in the problem solving domain is extensive. I will consider the sources that are more or less directly related to physics, although many authors of such studies contend that the skills, necessary for successful problem solving, manifest themselves (and, in fact, should be taught) across disciplines.

An interesting dichotomy can be found in physics-related literature. Some authors (mostly, cognitive psychologists and college physics professors) offer very general strategies that focus on imparting the general problem-solving cognitive skills. These authors (whom, for the purposes of this review, I would like to broadly call "thinking-skills" proponents) favor the "holistic" approach to problems and strongly caution against the over-prescribed "first, isolate the unknown" strategies, often presented in the textbooks. Such strategies are deemed novice-like by these authors.

However, another group of authors (mostly, high school physics teachers and, importantly, the textbook authors) advocate exactly that latter way of teaching. They like to present the problem solving procedures as sequences of well-defined steps ("first, isolate the unknown" and the like), so that the student just has to learn those sequences to successfully solve "any problem[1] in the textbook." For the purposes of this review, I would call such authors the "cookbook" proponents.

This dichotomy is certainly not perfect, as my examples will show; however, its existence is well-established. In my review, I will attempt to juxtapose the views of these two camps. Meanwhile, I urge the reader to look carefully at the examples of the tasks that the authors solve using their suggested strategies.

The "cookbook" strategies.

In this section, I will present the strategies offered by the practitioners who advocate what are undoubtedly novice-like approaches. However, the "expert" strategies, apparently, are not really needed for the typical textbook tasks (Heller & Hollabaugh, 1992; Lin, 1982) which makes the methods presented here seem appropriate for the high school teachers to teach and tempting for their students to adopt.

For instance, Stahl (1994) notes that many students claim understanding while admitting that they cannot solve problems. She fails, however, to note the fundamental misconception about the *nature of understanding* in physics that is held by such students (compare with Hammer, 1989). Stahl simply suggests that such students "may have *followed the teacher's steps*[2] in solving problems, but did not grasp the basis for choosing these steps. Their tendency… is to memorize the solutions to individual problems rather than master the general analysis applied flexibly to most of the problems they are assigned."

To rectify the problem, the author proposes her own seven-step "diagnostic procedure", which purports to be the "general analysis." In a way, it is, but some steps appear confusing. Consider, for instance, Step 2 "What information does the problem provide?" (too vague for a student to use) and Step 4 "Count… unknowns: we will need as many independent equations as the number of unknowns" (simply not true – besides, in a serious problem the extraneous variables may play a major role).

The problem, chosen by the author to illustrate her procedure, is admittedly "classical":

> "A proton ($q=1.60 \times 10^{-19}$ C; $m=1.66 \times 10^{-27}$ kg) is first accelerated from rest through a potential difference of 4.00×10^{6} V. It emerges through a small aperture at the center of the last electrode and moves in field-free space until it enters a region with a magnetic field B. It leaves the magnetic field region in a direction perpendicular to its entry direction. The radius of curvature is 2.00 m. Calculate B (magnitude and direction) and the length of time t that the proton spends in the magnetic field." (p. 466).

This is a typical end-of-chapter problem (poorly stated, too: either direction or the magnitude of B must be given).

Van Ausdal (1988) notes the low teaching effectiveness of problem-solving examples in the textbooks, noting how "a sequence of notational changes… in complex vector, delta or derivative notations" (p. 518) overwhelm the students. The author blames the lack of student understanding on the lack of "unifying

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[1] Any exercise, maybe…

[2] Italics are mine - B. K.

techniques" presented in these examples. He offers a "highly structured" method of problem-solving in kinematics as an example of a possible remedy. Eight steps are used, many of which I found either confusing or too narrow to be helpful. For instance: "Choose the Initial and Final Frames" (p. 519)) or "Is Any Portion of the Motion Constant Acceleration?" or "Count the Unknowns and Solve" (p. 519 - compare with Stahl's Step 4). The "friendly" comment "If there are no more than three unknowns, you can usually solve for everything" (p. 519) epitomizes, in my opinion, the novice-oriented frame of mind of the author. Fittingly, the problem chosen to illustrate the method hardly warrants any "steps" beyond "looking up the right formula":

> "A player throws a basketball vertically downward at a speed of 20 ms^{-1}. It bounces very high, remaining in the air for two seconds before it bounces again. How high did the ball go?" (p. 520).

Wood (1985) offers a "general" method that is even more advanced: it uses not seven, not even eight but *nine* steps! Here are some of them:

> "1. make a simple sketch showing all given quantities… relationships that use the given data are most useful.
> 2. Write the relationship as specifically as possible.
> 3. Try to choose the relationship that involves information stated in the problem
> …
> 7. In some cases the new quantity cannot be solved for directly. You may need to look for the unknowns simultaneously." (p. 32).

Wright & Williams (1986) note the students' tendency to "memorize equations and long procedures with little understanding… plug numbers into equations and produce answers." (p. 211). The authors advocate explicit teaching of problem-solving techniques and present an eight-step[3] process called WISE (p. 212). However, in my opinion, the authors' strategy promotes the very approaches they lament: for instance, "I" stands for "Isolate the unknown" and "S" for "Substitute" (as opposed to "plug"?). To be fair, "W" means "What is happening?", a three-step part whose main purpose, however, is finding "knowns and unknowns" (p. 212).

Here is an example of a problem that is solved in the article:

> "A box is pushed across the ground with a steady force of 100 N that is inclined at an angle 20^{o} below the horizontal. If the box has a mass of 5.0 kg, and the coefficient of kinetic friction is 0.20, what will the speed of the box be after 5.0 m of pushing, starting from rest?" (p. 213).

I would classify such "problem" as an exercise (perhaps, an advanced one) unless this is the students' very first exposure to Newton's laws. The strategy, described by the authors, does provide a somewhat useful structure for solving the *exercises*. However, it hardly helps the students learn to deal with unfamiliar situations: it does not create *mental disequilibrium* (Lawson & Wollman, 1975).

Teachers' dissatisfaction with the "cookbook" strategies is reflected by Padgett (1991). In her article, the author pulls together all the "steps" from various sources into a "union list", containing 33 (!) steps. The author notes "remarkably little duplication" (p. 238) across the sources. As for her own advice, Padgett suggests: "highlight small groups of problem-solving steps… and give… problems *carefully designed* [4]to illustrate their use." (p. 239).

Arons (1981) also gives a decisive rebuttal to all "cookbook" advocates:

> "It is blandly assumed that if the student can get the "answer" he must understand the physics. Socratic dialog reveals that this is most definitely *not* the case. It is essential to train the students to a fixed habit of asking themselves for a restatement of the meaning of every term they encounter in a problem… This habit can then guide them through new situations where help from the teacher is no longer available." (p. 172).

While I do share these feelings, my lingering question is this: why can't we offer the students problems such that the fact that the student "can get the answer" *would* mean that "the physics" is, in fact, understood?

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[3] The authors concede that the early version had twice as many steps.

[4] Italics are mine - B. K.

The “thinking skills” strategies**.**

The authors in this group view problem-solving differently. Instead of proposing specific steps that the students must follow, they advocate teaching the general skills helpful in solving challenging tasks. The writings are both research- and experience-based. In general, I find myself more in agreement with this camp – with reservations discussed later in this chapter.

Arons (1981) notes some aspects of critical thinking that are crucial to developing conceptual understanding and effective approaches to problem-solving. Among them are “repeated opportunity, in slightly differing situations, to trace the line of reasoning and articulate it in their own words” (p. 170), “recognizing what is *not* the case” (p. 170), and “making oneself go back to definitions” (p. 171). Arons advocates Socratic dialog as the means of achieving such thinking skills.

Brouwer (1973) advocates the “process approach” to problem solving. He offers five categories of skills necessary for problem solving:

I. Initiation.
II. Collection of data.
III. Processing of data.
IV. Conceptualization.
V. Openendedness.
(from p. 483).

The author elaborates on each of these steps and is quick to recognize that any such list must be an “oversimplification” of what it takes to solve a problem. Still, he argues, it is important to keep in mind the skills relevant to problem solving and demonstrate their usefulness to the students. The author then shares the experience of having offered the students a problem “for which they lacked the mathematical background necessary to obtain quantitatively correct results. Rather than apply a well-known mathematical formalism, students had to muster what little mathematical knowledge they had to apply to the problem.” (p. 484). [5]

The problem offered to the class was as follows:

> “to predict, quantitatively if possible, the effect of a headwind or tailwind on the angle at which one would have to aim the projectile in order to achieve maximum range. *The problem was purposely ill-defined in order to force the students to limit the problem and to choose a model of air resistance”*[6] (p. 484).

As the reader can see, the format and the level of sophistication of that problem are quite different from those in the examples used by the proponents of the “cookbook” method, rendering it obsolete. The author suggests that posing such problems and opening them to discussion provides the students with “perhaps [the] only opportunity to do some real problem solving and provides the hope that they in turn can foster in secondary students a better inquiry approach to science.” (p. 486).

A two-part research report by Heller and her colleagues (Heller, Keith, & Anderson, 1992; Heller & Hollabaugh, 1992) discusses the issue of teaching problem solving through cooperative grouping. The authors offer a five-step strategy, building on earlier research (Polya, 1948; Reif & Heller, 1982):

- Visualize the problem;
- Create the physics description;
- Plan a solution;
- Execute the plan;
- Check and evaluate.

Heller & Reif (1984) discuss a theoretical prescriptive model of generating problem descriptions in physics; the authors claim that such a model is helpful in subsequent problem-solving processes. To test the model, the authors used “problems in college-based physics, specifically in the field of mechanics. Problem solving in this domain is realistically complex, representative of other quantitative… fields, and often difficult for many students. It is also… a serious challenge in physics teaching.” (p. 181).

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[5] This approach resonates well with my view that a task, in order to qualify as a *problem*, must be *challenging in the context of prior knowledge.*

[6] Italics are mine - B. K.

The authors suggest that the initial problem description (representation) in terms of the applicable physics concepts is crucial in effective problem-solving. The study showed that the model, indeed, made the students more skillful problem-solvers in terms of the number of errors they made. The authors offer two examples of "blocks and pulleys" problems, which are "standard" in the sense that, once all forces acting on the objects are identified, the rest of the solution is pretty much in the algebraic domain. However, using the prescribed model, the students identified the direction of the forces better and also omitted fewer forces than the control group. The model may, indeed, be helpful in training the students to solve the "standard" tasks (exercises). However, its validity for "teaching students improved scientific problem-solving skills" (p. 207) remains to be seen: no challenging, "real-life" problems appear to have been offered to the participants.

Hestenes and his group have developed the teaching method whereby the students are focusing on "understanding the physical world *by constructing and using the scientific models* to describe, explain, predict, and to control physical phenomena." (Wells, Hestenes, & Swackhammer, 1995, p. 606). Students "learn transferable skills by applying given models to a variety of situations" (Hestenes, 1996, p. 1). As applied to problem-solving, the method calls for using "ill-defined", real-life settings for the problems used in instruction and the use of the most general physical principles ("scientific models") in approaching those problems. (See also Hestenes, 1987).

Larkin (1981) describes the computer problem-solving program that uses the strategies commonly attributed to the experts, such as generating sequential problem descriptions, in which the set of equations comes as the fourth step (unlike the novices, for whom writing equations is typically the first step of the solution). However, once again, the author's conclusions about what constitutes an effective problem-solving strategy are discounted in my mind by the triviality of problems that she used. The difficulty of the problems was chosen so that "*the skilled subjects would require at least a minute to solve them.*[7]" (p. 535).

Reif, Larkin, & Brackett (1976) discuss the structure of "expert skills" in problem solving and pose an important question: "How can a student be taught to develop the general skill for gaining an understanding of any new relation presented to him?" (p. 213). In my terms, how can the skill of *bisociation* be taught? The authors' solution is "systematic training… where [such] abilities are explicitly apparent and properly rewarded" (p. 213). The training program developed by the authors was then tested by using the random samples of 75-150 college students. The authors claim that after the explicit instruction on understanding new relationships, the students did substantially better in understanding new relationships than they did after traditional instruction. However, there does not appear to have been any testing of their actual problem-solving abilities in physics. Their suggested problem-solving strategy (Description – Planning – Implementation – Checking – from p. 216) is very similar to the one described in Polya (1948).

Lin (1979, 1982) discusses the dichotomy between "learning physics vs. passing courses" existing in college physics instruction. He notes that the natural reaction of the students is to satisfy the "hidden curriculum" (that is, to pass the course) which leads to their adoption of cognitively primitive problem-solving approaches. The author offers a number of excerpts from student self-described approaches:

> " - I look at all the variables and try to come up with an equation"
> - I try to find one equation that relates the givens and the unknowns"
> - If I don't know what I am doing…, I like to plug in the numbers and work it out from there"
> - Physics is equations"
> (pp. 152-153).

Lin suggests ways of aligning the long-term goals of instruction (instilling effective problem-solving strategies, in particular) with the short-term goals of the students (to pass the course). The author proposes explicitly stressing problem-solving skills during the instruction and the grading system that rewards coherent, thoughtful solutions - not just "the right answers."

Reif & Heller (1982) note that "(t)he most common method of teaching scientific problem solving … with sufficient examples and practice… is neither very efficient nor effective." (p. 124). "A potentially more effective instructional method would teach problem-solving skills *explicitly* on the basis of insights derived from a model of effective problem solving." (p. 124)[8]. The authors offer an example of a fairly challenging problem, which can be solved by *explicitly invoking* the problem-solving skills described:

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[7] Italics are mine – B. K.

[8] More discussion of explicit teaching of general learning and problem solving skills can be found in Reif, Larkin & Brackett (1976) and Reif (1981).

> “A pendulum consists of a small ball attached to one end of a light string of length L. The other end of the string is attached to a hook fastened to the ceiling. A fixed peg is located vertically below the hook at a distance smaller than L. The ball is initially held at rest, with the string taut and horizontal, and is then released. What must be the minimum distance between the hook and the peg so that the string is still taut when the ball reaches a point directly above the peg?” (p. 113).

Unlike many “textbook” problems, this one does call for applying the “expert-like” approach. For instance, before any equations can be written, a “why?” question has to be asked[9]. However, the authors do not present any empirical evidence showing the improvement of the students’ abilities to solve such tasks as a result of using the described teaching methods.

Sweller (1988; see also Sweller & Cooper, 1985) suggests that the conventional process of problem solving is an ineffective tool for teaching problem solving since it “requires a relatively large amount of cognitive processing capacity which is consequently unavailable for schema acquisition” (p. 257). The author, interestingly, suggests that “development of (problem-solving) expertise may be retarded by problem-solving” (p. 284).

Milsop (1979) offers ways to *evaluate* the students' problem solving skills with a checklist of twenty-four questions. The author believes that such evaluation may prevent the physics instruction from being “reduced merely to the teaching and testing of definitions, formulas and plug-in problems” (p. 121). However, these questions leave a lot to subjective judgment (“Did the student indicate a logical strategy in approaching the experimental situation?” (p. 121), and the author does not suggest any examples of activities that would stimulate the skills outlined in the checklists.

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[9] For instance: "Why *is* there a minimum distance?"