

### Results: student interviews.

Eight students, from two local high schools, volunteered to be interviewed. While most students had agreed to the interviews in principle, when they signed their consent forms, recruiting the students for the interviews proved challenging. The students were selected to represent a wide range of problem-solving abilities, as determined by their CyberTutor task performance. Therefore, the interviews had to be conducted closer to the end of the academic year, after the CyberTutor tasks were completed, and many students were reluctant to spend time on things “optional” at that point. My goal, meanwhile, was to interview the sub-sample that would be a reasonable representation of the entire sample.

Eventually, about fifteen students agreed to be interviewed; from this number, eight were selected. Six students were male, two were female. Five were taking AP Physics as a second-year course; for the other three, it was their first physics course. Five were enrolled in BC Calculus, two – an AB Calculus, and one – in a higher-level mathematics course. All students were quite successful in their math classes, earning at least a B+.

The table below represents the statistical description of the interviewees’ performance-related information. The last column of the table presents the corresponding mean values for the entire sample. The last row in the table represents a new variable, which I constructed in an attempt to account for the students’ intellectual persistence (or lack thereof).

The “score” for each problem was given as follows:

0 if the student did not solve the problem and gave up;

1 if the student did not solve the problem but was stopped due to the lack of time;

2 if the student solved the problem correctly

Therefore, the maximum possible “interview score” was 8.

Table 18. Statistical characteristics of the interviewees.

	Minimum	Maximum	Mean	Std. Deviation	ALL: Mean
SAT-Math score	710	800	751.25	35.229	748.02
MDT score	43	96	63.87	17.916	58.08
MDT-R score	70	100	87.50	12.817	84.69
MDT-B score	10	90	45.00	23.299	33.98
CT Problem solved	0	5	3.38	2.200	2.25
Incorrect attempts	1	23	9.13	7.338	8.38
Hints used	0	17	8.25	6.563	11.19
R-hints used	0	6	2.00	2.619	3.54
B-hints used	0	12	6.25	4.432	7.66
Interview Score	1	7	4.00	2.070	

As one can see, the SAT-Math score ranged from 710 to 800, with the mean value of 751 (that mean is 748 for the entire sample). The interviewees, in average, were somewhat stronger than the entire sample in their problem-solving characteristics, as defined by their MDT-B scores (45% vs. 34%) and the mean number of problems solved (3.38 vs. 2.25). The average interviewee also used fewer hints and made fewer incorrect attempts than the average participant. However, the differences are not striking, and the range of all scores – including the interview score – among the interviewees was wide. Overall, I feel confident that the interviewed sub-sample was reasonably adequate in representing the entire sample.

The interview process is described in greater detail in Chapter 2. In total, 32 think-aloud protocols were collected (each participant attempted four problems). The protocols were then transcribed and analyzed; students’ notes taken during the solving process were also used in my analysis. Below, I present the detailed summary of the protocols; the discussion of these results can be found in Chapter 4<sup>1</sup>.

<sup>1</sup> As I quote from the protocols, I use the actual first names of my participants: Alex, Alissa, Colin, Dino, Maria, Nathan, Ricky and Prashant. I will also use the problem codes according to Appendix F.

All participants seemed to be fairly highly motivated; they genuinely tried to solve each problem, typically spending most of the allotted time before giving up; in many cases I had to stop them due to the time constraints. In general, I feel that, during the interviews, the participants may have shown more motivation to solve the problems than during their unsupervised solving of the CyberTutor tasks. For instance, each participant, on more than one occasion, was forced to rethink the assumptions, check the answer, discard the useless facts and equations and, sometimes, start all over – all students seemed at ease with this part of the solving process.

Also, the participants appeared to have been well-trained in exercise-solving: with few exceptions, they had no difficulties recalling the basic facts and using the basic procedures if they wanted to use them. The participants also understood that, in general, the interview problems were *supposed* to be hard: they were not intimidated when they acknowledged that “these problems are nothing like the ones I have seen before!”

### Correlational analysis.

To gain additional perspective and in an attempt to corroborate the findings from the CyberTutor tasks, I explored the correlations of the interview score with the background and performance variables for the interviewees. The results of this analysis, somewhat surprising, are shown in Table 19.

Table 19. Correlational analysis of the interview performance: **Interview score** vs. background and performance variables.

	MDT score	MDTR score	MDTB score	CyberTutor Problems Solved	Incorrect Attempts	Hints	R-hints	B-hints
N	8	8	8	8	8	8	8	8
Pearson Correlation	.890	.754	.889	.063	-.273	.000	-.105	.062
Sig.(2-tailed)	.003	.031	.003	.883	.513	1.000	.804	.884

While the number of interviewees is very small, and any statistical results obtained for such a group must be treated with the utmost skepticism, some numbers are worth noting.

First of all, no significant correlations were found between the interview score and any of the background variables. Second, there were *extremely high and highly significant correlations* between the interview score and the performance on the multiple-choice test – and, in the meantime, *virtually no correlation* between the interview score and the variables related to the open-response CyberTutor tasks such as the number of problems solved, the number of incorrect attempts and the numbers of hints used. The possible meaning of such drastic difference in correlation may lie in the fact that all interviewees seemed to try their best during the interviews; the same cannot be guaranteed regarding their CyberTutor tasks.

Also, the analysis showed that the total score was highly correlated with the scores earned on the individual problems (R values ranged from 0.60 to 0.89); such correlations reaffirmed my confidence in the validity of the tasks with respect to their ability to probe the problem-solving skills consistently.

### Problem-solving approaches.

Several broad patterns of problem-solving behavior emerged from the interview protocols. Within these patterns, several smaller-scale approaches could be discerned. In this chapter, I do not intend to conduct a detailed analysis of my observations; this will be done in the next chapter. However, to describe the results, a certain structure must be given to the quotations from the interviews, which I present below. The structure, at this point, is intended just for the convenience of the reader, to give the detailed, organized description of the behaviors observed. In the next chapter, I will suggest the hierarchical taxonomy of these behaviors and discuss its relationship with previously existing and proposed theoretical frameworks.

As the first step of the solution, the students usually **drew the sketches and the free-body diagrams** of the situation; they labeled the known quantities, drew forces, labeled the objects in motions, etc.:

“I’m just drawing a sketch for myself, so I know what it is.”

(Alissa, 12-19)

“I have a nice picture here, but I guess I’ll redraw it.” (Ricky, 12-19)

“I’ll draw a nice picture of this.” (Ricky, 17-15)  
 “Mind if I name the pulleys, at all?” (Colin, 17-15)  
 “We know that the ball is moving in circular motion, so we do free-body diagram...”  
 (Alissa, 13-33);  
 “I’m thinking of free body diagrams, first...” (Dino, 5-18)  
 “Well, first thing I’m definitely going to be doing is free body diagrams.” (Colin, 17-15)  
 “I’m still trying to figure out how to set up these free body diagrams so that I get the right picture” (Nathan, 5-18)

Also, when the students first saw the problem, they attempted to **identify the physics concepts and equations relevant to the problem situation**; sometimes they indicated what concepts they thought were useful and sometimes just stated that they were looking for such concepts:

“I’m just doing some kinetic energy equations about how fast the ball would be falling.”  
 (Colin, 13-33)  
 “I guess we do need conservation of energy, because you need to know the speed to find the tension.” (Dino, 13-33)  
 “I guess I’m going to use Newton’s second law.” (Colin, 17-15)  
 “Yes, conservation of energy...” (Nathan, 13-33)  
 “Ok. For this problem, I know I’m gonna have to use torques...” (Nathan, 17-15)  
 This type of approaches to the problem is to go with energy conservation, but because this is looking for acceleration, I... Uh, torque, and...” (Nathan, 13-33)  
 “The first thing that I’m thinking is conservation of energy, at the top it’s going to have all potential energy, gravitational potential, and I know at the bottom it will be all kinetic...” (Alex, 13-33)

All problems were in Mechanics, and the students found it important to **visualize the character of motion**:

“And we know when this is swinging, sort of like a pendulum motion, it is in circular motion.” (Alissa, 13-33)  
 “... I’m trying to visualize exactly what’s going on, it’s... try doing it after... accelerate together... trying to figure out which way the frictional force will act on the two blocks...”  
 (Nathan, 5-18)  
 “Let’s say this pulley goes up. Um. Ok. So, if this pulley goes...up...by a certain ...distance...wait. Ok. So...right, if we have this going up by a certain amount, this going down by a certain amount. Ok. Let me draw a better picture here.” (Ricky, 17-15)  
 “It’s really hard to figure out what this will actually do...because...all the strings are going all over the place” (Ricky, 17-15)  
 “So the simplest thing to do here is to figure out how the entire system would move first. Just get an intuitive grasp of what would happen if it were released.” (Prashant, 17-15)  
 “The first thing I’m doing is something I do with most mechanics problems that I unfortunately can’t do with E and M. I just try to kind of imagine what’s going to happen, um, as if I have the system, um, just, in front of me, and, and it’s released from rest, and I’m...just so I have an idea...”  
 (Alex, 17-15)

**Searching for special relationships.** Once the broad concepts were identified – and the corresponding equations written, the students looked for “missing equations” by searching for the relationships and ideas, specific to the problem, that could be translated into equations. This was, indeed, the key part of each solution: to notice something that makes the problem unusual, tricky, challenging. The students **announced the relationships that they discovered**:

“And we know that the acceleration in the y-direction will be zero, so that the net acceleration is horizontal” (Alissa, 13-33);  
 “Um, and we know the friction acts equally, um, equal in magnitude but opposite in direction on mass 1 and mass 2.” (Alissa, 5-18);  
 “Direction of velocity is always tangent to the direction of motion whatever point it is.”  
 (Alissa, 13-33);

“And the acceleration of 1 with respect to the ground is equal to the acceleration of 1 with respect to the pulley, plus the, um, the acceleration of the pulley, with respect to the ground...” (Alissa, 17-15)

“...we know is that because the prisms don't move with respect to each other when they are accelerating in the horizontal direction, their accelerations are equal...” (Alissa, 5-18)

“And, now, um...um, so they're not moving with respect to each other. Um, so the acceleration is  $F$  over  $2M$ . So that's gonna be the acceleration of the whole system.” (Dino, 5-18)

“These forces are all, most of them are going to have to balance, except for the left, which it should be accelerating, left, the whole system.” (Colin, 5-18)

Sometimes the students indicated that they were **searching for the special relationships but had difficulties identifying them**:

“The only way I know of going about this is using force, but if I'm given the period of rotation, I don't know how I can use that...” (Nathan, 12-19)

“I'm thinking...how can I relate the tension 1, which I decided was equal to tension 2, to tension 3, which is on the second mass?” (Nathan, 17-15)

“And it must mean something that the table is frictionless, and...It can't serve to simplify the problem any more than it already is.”

(Prashant, 5-18)

“I've got 3 pulleys to consider... Now, they're connected in a very strange fashion...”

(Prashant, 17-15)

“Now, you know  $L$ , you don't know  $V$ , and you have to relate it to  $\theta$ .” (Colin, 12-19)

Sometimes, in frustration, the students **indicated that they felt they could not find “the missing piece of the puzzle”**:

It's just that I've never seen, um, anything set up like this, with, um, I've never done anything with pulley actually, um, the pulleys actually moving, and I, I just don't know how to treat it.” (Dino, 17-15)

“Lot's of things to think about. What's interesting about this is, that, the pulleys are going to be accelerating at a certain rate, but they themselves are massless, and they have forces acting upon them. So, it's sort of odd.”

(Colin, 17-15)

“I don't like this condition here, about the accelerations, just because... it doesn't seem like a very nice relation. It's... so unintuitive, that, like, I don't understand where exactly that comes in... if  $a_3$  doesn't appear anywhere else, then it must be... really weird.” (Ricky, 17-15)

“I'm not really sure that I can predict the motion of the pulleys and the masses, and since I can't do that, I can't really seem to, uh, I can't really find a relationship for any forces that are involved.” (Prashant, 17-15)

“...I'm just not seeing it... Something's not working here... I don't think I'm going to get this...” (Alex, 13-33)

“I'm missing something very fundamental about this. I don't...understand quite what.”

(Nathan, 17-15)

**Anticipating future actions and needs.** The difference between “anticipating” and “searching”, as I see it, was in that “anticipation” included discussing the steps and procedures that would be needed not at the moment, but a few steps later. “Anticipation” was different from “searching” in the same way the act of trying to find a place to cross the river differs from the act of thinking: “Just *what* will I find there once I cross?”

I discerned several approaches related to “anticipation”:

First of all, the students often seemed to do **short-term planning**, discussing actions that may be needed or what particular equation will have to be invoked.

“I'll figure that out later...”(Dino, 12-19)

“Plugging in the original equation but I don't think that will get me very far. Let's try it.”

(Nathan, 12-19)

“So, some of these tensions are going to end up being, like, the same as each other or sums of each other, and things like that. But, I guess I’ll, figure that out later.” (Ricky, 17-15)

“I have a feeling that’s...we’re gonna need something else, like...yeah. Because...yeah. So. But we’ll see... So, let’s see, this is going to be...slightly ugly, anyway.” (Ricky, 17-15)

“we know that frictional force and the normal force have to be the same...which, I guess makes some sense, but we’ll see about that later.” (Ricky, 5-18)

“It seems that, um, I have what I need to find, since I am given the applied force, and I know the angle, I know that it has a mass, I know the force of the gravity, so using just the first equation, I can just find the force of static friction, and I would have to consider the uh, the, the equation consisting of the forces that aren’t perpendicular at all, but it seems... It seems too easy if it’s done that way.” (Prashant, 5-18)

“If I were to do that, then I’d just get a very, very complicated equation... well, maybe not that complicated. But it would be circular, it would just end up, just having one equation with 3 variables that I don’t know...”

(Prashant, 13-33)

“I can find what either the time will be, or what the displacement will be, and either way I can find the angle, it’s just a matter of converting linear to rotational, but... I have to go about finding V first.” (Prashant, 13-33)

“I could use that to find the angle because, well, if I could find V and R, which, neither of which I know, then I can find the angle, because I can just use this component of acceleration to define theta...” (Prashant, 13-33)

“If m1 is going to be going up or down or m2 is going to be going up and down so I can ... check my answers at the end and see if they make sense.” (Alex, 17-15)

Another approach that was, I felt, associated with anticipation was **questioning the relevance of knowledge**. The students understood that some facts, equations and relationships may be entirely correct – but irrelevant or not helpful to the solving process. As the students tried to select the useful concepts and equations, they often discard the presumably correct relationships and equations if they were not, in the students’ opinion, useful in solving the problem:

“I’ll always know that the normal force here is equal to the force of gravity. So there’s not particularly any reason to consider that.” (Prashant, 5-18)

“...thinking about getting a coefficient, but...but it doesn’t, it doesn’t even answer it.” (Dino, 5-18)

“Even if I do solve...the...question...will I have something that will give me both accelerations?” (Nathan, 17-15)

“I still don’t know the radius and whether or not I need it and...”

(Prashant, 13-33)

“I’m not sure that the formula that I have for the period applies. There’s something missing from this formula, or something that prevents me from turning it into something useful...” (Prashant, 12-19)

“Yeah. I don’t think that, uh, just defining the MV squared over R is useful because, I don’t know the, I don’t know the speed of, the square of the speed of the mass.” (Prashant, 12-19)

“... I remember that T is  $2\pi \sqrt{l/g}$ ... I think that is not for this situation... this is for a normal pendulum, not the three-dimensional, in this one you are going to have some circular motion, for which this equation would not count. I am not going to use that equation.” (Alex, 12-19)

**Reconstructing missing knowledge:** Most of the time, the students did remember the necessary basic equations, such as Newton’s laws, formulas of kinetic and potential energy, trigonometric equations, etc. However, if a particular equation was forgotten, the students attempted to create it using whatever equations and facts they did remember. If unsure about the correctness of a semi-forgotten equation, the students often checked it by juxtaposing with the knowledge they were certain about. One problem caused almost all instances of such “reconstructing”:

“...period is equal to total distance divided by the speed...of the... object around the circle, I believe. Yeah, that, that seems right. Yeah. *Unclear*. Velocity times time equals distance, so time equals distance over velocity, yeah...”

(Colin, 12-19)

“Trying to remember what I can about the pendulums and the equations we use for them... Trying to remember what we learned about the pendulums.. probably in November... about the simple pendulum, physical pendulum, torsional pendulum...” (Alex, 12-19)

“Here is the rotation and I’m trying to drudge up the knowledge I remember of... I can use that...” (Nathan, 12-19)

“T is the time it takes to go that distance, then I can find the speed. Speed is just a function of, uh, speed is just a function of distance over time. The speed would be... The constant speed would be equal to, uh,  $2\pi R$  over T, and if that’s the case then I can redefine T as...  $2\pi$  over V, yeah, because, time equals distance over speed.” (Prashant, 12-19)

“I don’t remember... I have not done this for quite a while... ummm... I am going to try to get T in terms of omega and  $2\pi R$  or something like that.” (Alex, 12-19)

One of the common and specific cases of anticipation, as I see it, was **introduction of new variables**. The students introduced new variables when necessary; they expressed hope that the new unknowns would either be found from the equations or cancel out at some point in the process of solving equations.

“I assume that this has a length L, just...out of curiosity...” (Colin, 13-33)

“So, um...so we have to assume that it has a speed V, rotating, V...” (Colin, 13-33)

“So, uh, we’ll call acceleration of this,  $a_3$ ...” (Ricky, 17-15)

“Ok, it doesn’t give us a lot of stuff, but that’s probably cause it’ll cancel out or something later on. But, uh, I’ll just put in Mass M, and,...L, and...ok.” (Ricky, 17-15)

“Ok, so it’ll probably cancel out sometime later...” (Colin, 13-33)

As an extreme case of anticipating, I noticed a frequent **use of intuition** when the students declared that, despite the lack of proof or justification, they wanted to pursue a particular approach because they just “feel right” about it.

“I can try it...my initial instinct that it would push it... up” (Alissa, 5-18)

“My guess is that it’s going to be some sort of...sinusoidal function.”

(Dino, 13-33)

“I’m kind of taking a stab...” (Dino, 13-33)

“I’m unsure... I don’t know exactly... end up going with my gut instinct...”

(Nathan, 5-18)

“I am just going to go on my gut feeling which is that that’s correct... I think that ... maybe that is wrong...I really don’t know but it does not seem right...” (Alex, 12-19)

“... my gut impulse is to say when it’s at the bottom and the angle’s zero, but I know that’s probably wrong...” (Alex, 13-33)

When the students could not grasp the given situation, they sometimes tried to **look for special cases** or **made simplifying assumptions** that made the situation easier to analyze and would then attempt to transfer their understanding into the actual problem situation.

“Um...find a point halfway through...so, I’ll do a 45 degree angle one.” (Dino, 13-33)

“If  $m_1$  was really heavy,  $m_1$  would go down and  $m_2$  would go up. If  $m_2$  was really heavy,  $m_1$  would go up and  $m_2$  would go down.” (Dino, 17-15)

“I’m just going to pretend there is no mass 2, and just ignore that pulley for now to see how this works.” (Dino, 17-15)

“Ok, so, what do we know? Ok, I guess, looking at, each of the pulleys first would make the most sense.” (Ricky, 17-15)

“I’m going to assume that there’s no tension in the string, right when the ball...” (Nathan, 13-33)

“...what if I were to forget about acceleration for a minute and say that the system is not moving...and then find out the tensions” (Alex, 17-15)

**Executing.** Once the necessary equations (all of them or just some of them) were obtained, the students would solve them – *execute* the solution. The participants took for granted that sophisticated problems may require the use of multiple equations and were not reluctant to introduce new equations as needed. The algebra skills of the participants were seriously tested: I found that, although the students were good at

performing mathematical procedures correctly, they were often at a loss trying to solve the simultaneous equations in the most rational way. Often, the students arrived at spectacularly ugly mathematical constructions and skillfully (but pointlessly) fought them for a long time before deciding to try an alternative approach.

As the students looked for special relationships within the problem, attempted to translate such relationships into equation, solved those equations, looked for more equations, etc., they used a wide range of **self-monitoring** (or self-regulation, as Lawson & Wollman (1975) describe it) techniques. These techniques included:

**Pausing for general review and checking.** It was interesting to observe the students pacing themselves through lengthy, intellectually challenging solving processes. The students often paused to conduct a “general review” of their progress. Such reviews were sometimes prompted by mounting mathematical difficulties or the obvious lack of equations. However, the students also paused for reviewing just to make sure that they were about to take “the right turn” before going too far:

“I’m just making sure I didn’t miss anything... Well, they do have the same mass. That makes things easier, just. I feel sort of dumb.” (Colin, 5-18)

“Um, I’m checking my equations to make sure that I didn’t leave anything out, and to make sure that I used both components of the friction...” (Alissa, 5-18)

“Let me think if there’s any way I can come up with something better for that.” (Dino, 13-33)

“Um, well, you know the free body diagram is correct, because those are the only two things touching it.” (Colin, 13-33)

“Now, let’s see if that makes any sense. One...one qualm I have about that...is that pulley 1 is connected to M1...” (Colin, 17-15)

“Well, right now I’m trying to think of why the frictional force is not equal to  $F \cos \theta$ . Second-guessing myself.” (Nathan, 5-18)

“Ok, it makes slightly more sense, I guess, although not really. So, where were we before? ... So, let’s go back to these equations and see what we have here.” (Ricky, 17-15)

“I’d really like to say some of these are the same, but, I don’t know which ones would be the same.” (Ricky, 17-15)

“... every single expression either points to that there’s no acceleration at all, or the acceleration is just the acceleration due to gravity.”<sup>2</sup>

“I am just thinking how that would affect my other equations and if that makes sense or not.” (Alex, 12-19)

“And, I’m just trying to justify that in my mind, to make sure that that’s correct.” (Alex, 17-15)

“I think probably I screwed up somewhere, but, um... like, the hypotenuse is going to be  $mg \cos \theta$  over cosine theta... I’m almost positive I messed up somewhere...” (Alex, 13-33)

“Now I’m wondering if I want, if I should take the components like I did, or if I should not have taken the components because friction is going to be acting either up or down the plane, so I’m going to need something to set that equal to...” (Alex, 5-18)

**Recognizing and discarding useless or wrong results:** in general, the students were ready to recognize their errors and attempt to correct them.

“Ok, I think I made a mistake, because, um, I did not account for the length of the pendulum, which affects, um, the centripetal acceleration and also the relationship of velocity to period.” (Alissa, 12-19)

“I thought that this entire time I was trying to find the force that was being applied to both blocks, but...they’re prisms...whatnot. That makes my life, a lot easier.” (Colin, 5-18)

“I believe. So, that’s where I went wrong. Before I had just assumed...well, let’s see. This, I said it was  $2MG \cos \theta$ . Um...work that back out ...” (Colin, 13-33)

“Oh, sort of a circular equation, isn’t it? So, let me try again, and see.” (Colin, 5-18)

<sup>2</sup> This was one of the two occasions when a student, in passing, mentioned the correct answer ( $a=g$ ) but did not focus on it, assuming it absurd.

“So...oh, I misread the question. So, um, I guess that’s, all right. This one’s still going to be right. And. Um. One of these is going to change. This one is going to change.” (Ricky, 5-18)  
 “I’d like to say they were the same. But if they were, then that would mean that they would all be zero, and that wouldn’t really, make any sense, because then the whole system would just fall apart.” (Ricky, 17-15)<sup>3</sup>

**Considering alternative approach.** Sometimes, especially after failing to arrive at the solution through a particular approach, the students would, rather than correcting the equations, abandon a certain line of reasoning or the use of a particular concept and “back up” to explore the alternatives:

“Ok, I’m just gonna just rewrite this. And think of something else. Ok. Cause, this isn’t going to be nearly enough to do this, I don’t think.” (Ricky, 17-15)

“But, that doesn’t work. So, I can try it, switching...” (Alissa, 5-18)

“So, I’m thinking of either trying something completely different, or trying to go with this some more. So, I have to think about what else there is.” (Ricky, 17-15)

“Uh, if I go with my, uh, second hypothesis that the pulleys will eventually align and reach the same height and stop there, then...” (Prashant, 17-15)

“Let’s get back to where I started.” (Nathan, 17-15)

“...so I right now thinking that I am going to solve it up both ways and come up with two different answers: one is if one equation is right and the other is if the other equation is right ...” (Alex, 12-19)

“... I am going back to the beginning and thinking that it is only going to be  $F \sin \theta$  that is going to be equal to  $mv^2$ , because that is the only one that is in the plane of circular motion...” (Alex, 12-19)

“Going to forget that for right now, and I’m going to go back to what I had on the first block and say that...” (Alex, 5-18)

**Checking the final answer:** sometimes, the students checked the correctness of the final answer before presenting it to me:

“T gets bigger, that means that something should get bigger, something should get smaller.

*Unclear.* This is sort of weird. *Unclear.* This is...ok, well, we know T over...ok...so this...looks like...it sort of makes sense. Because...if you have...sort of saying... *unclear*...let me go back and check it.” (Ricky, 12-19)

#### The dogs that did not bark.

There were several strategies that have long been recognized by the educators as useful in the solving process – yet the students never employed those strategies.

**No long term-planning.** As mentioned above, when presented with a problem, almost immediately the students felt compelled to write some equations: sketching a free-body diagram and searching for relevant relationships obviously served the purpose of writing equation. At no time did any of the students attempt to map out the entire solution path before executing it, even in the most general sense. What I earlier described as “anticipating” *never had the final answer in sight*; it always seemed to served to facilitate the current step of the solving process, whatever it was at the moment.

**No attempts to use the “backward” solution strategy.** The students always used the “forward” strategy of solving: from known to unknown. Such strategies are commonly attributed to the experts in the literature (Larkin, McDermott, Simon, & Simon, 1980; Mayer, 1990).

None of the students, so far as I could notice, tried to *first* construct the equation that would contain the target unknown along with some other, possibly unknown, terms and then create additional equations to eliminate these “unneeded unknowns.”

**No attempts to use dimensional analysis.** While this appears to be a less fundamental omission, it is noteworthy that none of the students ever used dimensional analysis to construct the answer, to obtain a missing equation or to check the correctness of the obtained answer.

<sup>3</sup> Interestingly, that was the other moment any of my eight interviewees ever came close to solving 17-15; the correct answer is, in fact, “both masses will fall freely, and the system will ‘fall apart.’” However, Ricky did not recognize this possibility and discarded his hypothesis, eventually failing to solve the problem.