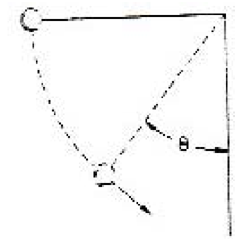
Appendix F

Problems used for student interviews.
All problems are supplied with *brief* solutions.

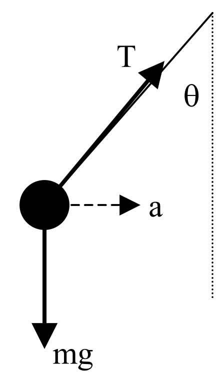
Problem 13-33

13. A small ball hangs from a string. The string is brought to a horizontal position and then released. At what point would the net acceleration of the ball be directed horizontally?



Solution outline.

Denoting the mass of the ball as m and the tension in the string as T, we can write Newton's 2nd law for two different directions:



Horizontal:

$T \sin \theta = ma$

where a is the (horizontal) acceleration of the ball.

Vertical:

$T \cos - mg = 0$

since the acceleration has no horizontal component

Also, the acceleration along the string is the centripetal acceleration; from the geometry of the situation, one can write:

$a_c = a \sin \theta = v^2/L$

where L is the length of the string

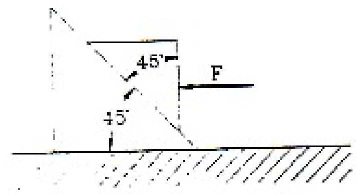
Finally, from the law of conservation of energy

$mv^2/2 = mgL \cos \theta$

Combining these equations leads to the answer: $\boxed{\theta = \tan^{-1} \sqrt{2}}$

Problem 5-18

5. A prism of mass m and with an angle of $\theta = 45^o$ (see the diagram) is located on a frictionless horizontal table. Another prism with the same mass sits on the first, bigger prism as shown and undergoes a force F applied horizontally. It is known that the prisms do not move with respect to each other. Find the force of friction F_f between the prisms.



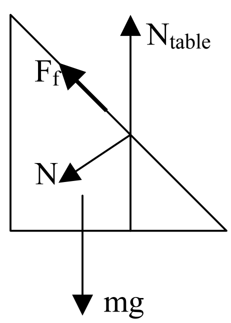
Solution outline.

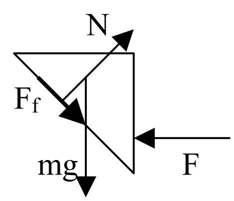
Since the prisms do not move relative to each other, they can be considered as one object whose acceleration can be easily found from Newton's 2nd law:

F = 2ma, so a=F/2m

According to Newton's 3rd law, each prism exerts two forces on the other: the normal force of magnitude N and the force of friction of magnitude F_f. We can assume that the force of friction acting on the bottom prism is directed upward. Now consider Newton's 2nd law for each prism separately (all components are in the direction of force F). For brevity, we will use the fact that $\sin 45^o = \cos 45^o = \frac{\sqrt{2}}{2}$

For the bottom prism:





$$(N + F_f)\frac{\sqrt{2}}{2} = ma$$

For the top prism:

$$F - (N + F_f)\frac{\sqrt{2}}{2} = ma$$

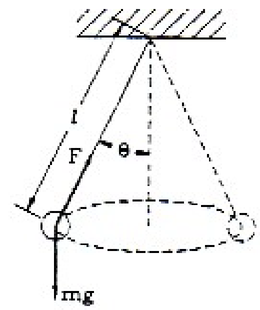
In addition, we can use the fact that the small prism has no vertical acceleration:

$$mg + (F_f - N)\frac{\sqrt{2}}{2} = 0$$

Combining the equations, we can obtain the answer: $\boxed{F_f = (0.5F - mg)\frac{\sqrt{2}}{2}}$

Problem 12-19

12. A simple pendulum is rotating in a horizontal plane hanging from a string of length L. The period of rotations T is given. Find the angle θ the string makes with the vertical.



Solution outline.

It is important to understand that the radius of rotation is not equal to L:

$r = L \sin\theta$

The period of rotation T us related to velocity as

$T = 2\pi r/v$

where v is the linear speed of the ball.

Also, since the ball has no vertical acceleration, from Newton's 2nd law:

$F \cos\theta = mg$

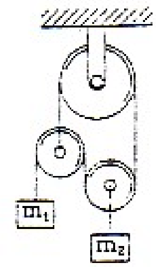
In the horizontal direction, Newton's 2nd law for the ball gives

$F \sin\theta = mv^2/r$

Combining the equations gives the answer: $\boxed{\theta = \cos^{-1}(T^2g/4\pi^2L)}$

Problem 17-15

17. Find the accelerations of the blocks m_1 and m_2 in the system shown. Assume the pieces of the strings not touching the pulleys are vertical.



Solution outline.

The string attached to block m_1 is then wrapped around each of the two movable pulleys and then goes around the top pulley and comes back to the left movable pulley. The tension in that string is constant throughout, since the string is presumed massless[1]. This tension can be denoted as T.
Each pulley is also presumed massless[2]; therefore, the net force acting on each pulley has to be zero. If one considers the left movable pulley, it is clear it is being pulled up by force T and is being pulled down by *two* forces T. Since the net force on the pulley must be zero, it follows that T=2T, and T=0. Therefore, there is no tension in the string attached to m_1, and both blocks are in free fall: $\boxed{a_1 = a_2 = g}$.

[1] This is not explicitly stated in the problem, but it was made clear to each student from the onset of the interview.
[2] See the previous footnote.